Integrating Kinetic Effects in Fluid Models for Magnetic Reconnection

Liang Wang[1] Ammar Hakim[2] Amitava Bhattacharjee[1,2]

[1]University of New Hampshire, Durham, NH USA

[2]Princeton Plasma Physics Laboratory, Princeton, NJ USA

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Outline

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Breaking the Frozen-Flux Constraint

- In a collisionless plasma, the violation of the frozen-flux constraint can be parameterized by the generalized Ohm's law:

$$\mathbf{E}+\mathbf{u}\times\mathbf{B}=\frac{\mathbf{J}\times\mathbf{B}}{n|e|}-\frac{\nabla\cdot\mathbf{P}_e}{n|e|}+\frac{m_e}{n|e|^2}\left[\frac{\partial\mathbf{J}}{\partial t}+\nabla\cdot\left(\mathbf{uJ}+\mathbf{Ju}-\frac{\mathbf{JJ}}{n|e|}\right)\right]$$

- We focus on the electron pressure gradient term, $-\nabla\cdot\mathbf{P}_e/n|e|$
  - Electron pressure *tensor* is defined by

$$\mathbf{P}_e\equiv m_e\int(\mathbf{v}-\mathbf{u}_e)(\mathbf{v}-\mathbf{u}_e)f_e(\mathbf{x},\mathbf{v},t)\,d^3v=\begin{bmatrix} P_{xx,e}, & P_{xy,e}, & P_{xz,e}, \\ P_{xy,e}, & P_{yy,e}, & P_{yz,e}, \\ P_{xz,e}, & P_{yz,e}, & P_{zz,e}, \end{bmatrix}$$

  - $\mathbf{P}_e$ is *anisotropic* if its diagonal elements are not equal
  - $\mathbf{P}_e$ is *non-gyrotropic* if its off-diagonal elements are nonzero
  - a *scalar* pressure, $\mathbf{P}_e=p_e\hat{\mathbf{I}}$, is isotropic and gyrotropic

Collisionless Reconnection Needs Full $\mathbf{P}_e$ Tensor

- Fully kinetic simulations of 2D, no guide-field reconnection indicate that at reconnection site,
  - $\mathbf{P}_e$ is indeed anisotropic and non-gyrotropic
  - $-\nabla \cdot \mathbf{P}_{e,\mathrm{off}}/n|e|^2$ is the dominant source of E_{rec} in Ohm's law
- With a guide field, the dominance of $-\nabla \cdot \mathbf{P}_{e,\mathrm{off}}/n|e|^2$ is somewhat weakened, but it is still important
- This term is seen to be important in electron diffusion region (EDR) crossings in Cluster data
- It is also expected to be important for MMS studies of EDR

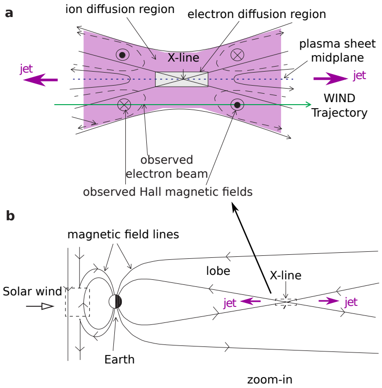

Figure : Adapted from Ø ieroset et al. (2001)

Previous Work

Towards Integration of Full $\mathbf{P}_e$ in Fluid-Based Models

Motivation

- Fully kinetic models containing full $\mathbf{P}_e$ tensor are usually infeasibly slow for large scale problems like Earth's magnetosphere
- It is necessary to parameterize anisotropic, non-gyrotropic effects in reduced, fluid-based models

Previous Works

- Hesse and Winske (1993) and Yin et al. (2001): Hybrid or Hall MHD with $\partial_t \mathbf{P}_e + \cdots = (\Omega_{ce}/\tau)\left(\mathbf{P}_e - p_e \hat{\mathbf{I}}\right)$ and vanishing heat flux
  - Advantages: Captures certain qualitative features of $\mathbf{P}_e$
  - Deficiencies: Not self-consistent; The "relaxation" term is not well-justified, and is not obvious how to improve
- Le et al. (2009): Equation of State for $p_{||}$ and $p_{\perp}$
  - Deficiencies: No off-diagonal elements; Fails when $|B| = 0$ or is small

Other Challenges in Global Simulations

Beyond the needs for full $\mathbf{P}_e$

Existing codes for Earth's magnetosphere have further deficiencies:

- Operates on uncontrolled numerical resistivity, but the real magnetosphere is highly collisionless
- Efficient implementation of the Hall term is difficult
  - Complicated implicit algorithms or artificial hyperresistivity
- Multi-ion species handling

We propose the multi-fluid moment model to address these issues along with the need for full P_e *tensor.*

Outline

Multi-Fluid Moment Model

Starting point, Vlasov equation for each species, s:

$$\partial_t f_s + \mathbf{v}\cdot\nabla f_s + (q_s/m_s)(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_v f_s = 0, \tag{1}$$

where $f_s(\mathbf{x},\mathbf{v},t)$ is the phase space distribution function

Basic idea, Take velocity moments of Eqn. (1), i.e., $\int \mathbf{v}^n f_s d^3v$:

$0^{\text{th}}:\ \partial_t n_s + \nabla\cdot(n_s\mathbf{u}_s) = 0,\ n_s \equiv \int f_s d^3v,$
$1^{\text{st}}:\ n_s m_s(\partial_t \mathbf{u}_s + \mathbf{u}_s\cdot\nabla\mathbf{u}_s) + \nabla\cdot\mathbf{P}_s = n_s q_s(\mathbf{E}+\mathbf{u}_s\times\mathbf{B}), \mathbf{u}_s \equiv \int \mathbf{v} f_s d^3v/n_s,$
$\vdots$

- Extensible to any order of moments and any number of species
- Always needs a closure at highest order
- Coupled to *full* Maxwell equations:
  $\partial_t\mathbf{B} = -\nabla\times\mathbf{E}, \partial_t\mathbf{E} = c^2\nabla\times\mathbf{B} - \sum_s n_s q_s \mathbf{u}_s/\varepsilon_0$
- No need to explicitly solve Ohm's law (embedded in $\partial_t\mathbf{u}_s$ equations)!

10- and 5-Moment Limits of the Multi-Fluid Model

10-moment model (calculating full pressure tensors)

- Truncate at $\mathbf{v}^2$ moment, omitting subscript $_s$ for species kind,

$$\partial_t P_{ij} + u_m \partial_m P_{ij} + P_{ij} \partial_m u_m + \partial_m u_{[i} P_{j]m} + \partial_m Q_{ijm} = (q/m) B_l \varepsilon_{ml[i} P_{jm]},$$

  where $\mathbf{Q} \equiv m \int (\mathbf{v} - \mathbf{u})^3 f d^3 v$ is heat flux
- A closure is needed to determine $\partial_m Q_{ijm}$

5-moment model (calculating scalar pressures)

- Assume $\mathbf{P}$ is gyrotropic and $\partial_m Q_{ijm} = 0$, the equations can be closed by

$$\partial_t \mathscr{E} + \partial_m [u_m (p + \mathscr{E})] = 0$$

  where $\mathscr{E} \equiv 3p/2 + mnu^2/2$ is the total fluid energy
- Equivalent to adiabatic Equation of State, $\partial_t p + \mathbf{u} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{u}$, with $\gamma = 5/3$

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5-Moment vs. Hall MHD

Is 5-moment consistent with the widely used Hall MHD?

5-moment equations

$$\partial_t n_s + \nabla\cdot(n_s\mathbf{u}_s) = 0, s = e, i$$
$$\partial_t \mathbf{u}_s + \mathbf{u}_s\cdot\nabla\mathbf{u}_s + \nabla p_s/m_s n_s = (q_s/m_s)(\mathbf{E}+\mathbf{u}_s\times\mathbf{B})$$
$$\partial_t\mathbf{B} = -\nabla\times\mathbf{E}, \partial_t\mathbf{E} = c^2\nabla\times\mathbf{B} - \sum_s n_s q_s\mathbf{u}_s/\varepsilon_0$$

Hall MHD equations

$$\partial_t n + \nabla\cdot(n\mathbf{u}) = 0$$
$$\partial_t\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p/mn = (|e|/m)(\mathbf{E}+\mathbf{u}\times\mathbf{B})$$
$$\mathbf{E}+\mathbf{u}\times\mathbf{B} = \frac{\mathbf{J}\times\mathbf{B}}{n|e|} - \frac{\nabla p_e}{n|e|} + \frac{m_e}{n|e|^2}\left(\frac{\partial\mathbf{J}}{\partial t}\cdots\right)$$
$$\partial_t\mathbf{B} = -\nabla\times\mathbf{E}, \nabla\times\mathbf{B} = \mu_0\mathbf{J}$$

- 5-moment and Hall MHD both retain Hall term and scalar pressures.
- 5-moment equations can formally reduce to Hall MHD equations with the following assumptions:

Eqn., 5-moment	Assumption	Eqn., Hall MHD	Physical implication
$(1/c^2)\partial_t\mathbf{E} = \nabla\times\mathbf{B} - \mu_0\mathbf{J}$	$\varepsilon_0\to 0$	$\nabla\times\mathbf{B} = \mu_0\mathbf{J}$	$c\to\infty$, no EM waves
$\nabla\cdot\mathbf{E} = \|e\|(n_i - n_e)/\varepsilon_0$	$\varepsilon_0\to 0$	$n_e = n_i = n$	neutrality, $\lambda_{\mathrm{Debye}}\to 0$
$\partial_t\mathbf{u}_s + \cdots = \frac{q_s}{m_s}(\mathbf{E}+\mathbf{u}_s\times\mathbf{B})$	omit $O(m_e/m_i)$	$\mathbf{E}+\mathbf{u}\times\mathbf{B} = \cdots$	$\omega_{pe}\to 0$

- The effect of m_e is also confirmed numerically (not presented here).

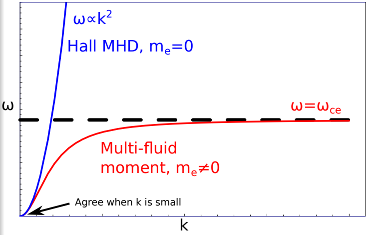
Tim Step and Grid Size Constraints

Whistler dispersion is not artificially quadratic!

- $\Delta t_{\mathrm{wh}} = \mathrm{CFL} \cdot \Delta x / u_{\mathrm{wh}}$, where $u_{\mathrm{wh}} = d\omega/dk$
- When $k \to \infty$
  - Hall MHD: $\omega \propto k^2 \to \infty \Rightarrow \Delta t_{\mathrm{wh}} \propto k^{-1} \to 0$
  - Multi-fluid moment: $\omega \to \omega_{ce} \Rightarrow \Delta t_{\mathrm{wh}}$ is not limited as a result of $m_e \partial_t \mathbf{u}_e \neq 0$

The well-known constraint on explicit time step in Hall MHD is eliminated!

Dispersion relation of whistler waves



New constraints introduced? No problem

- No need to resolve λ_{Debye} and ω_{pe} if using a simple locally implicit algorithm
  - The basic idea: implicit solving using data in a single cell, not global matrix inversion coupling all cells
- The CFL constraint due to speed of light still exists, but is relatively less severe

Outline

Construct Collisionless Closures: Hammett-Perkins Approach

A crucial step to correct treatment of $\mathbf{P}_e$

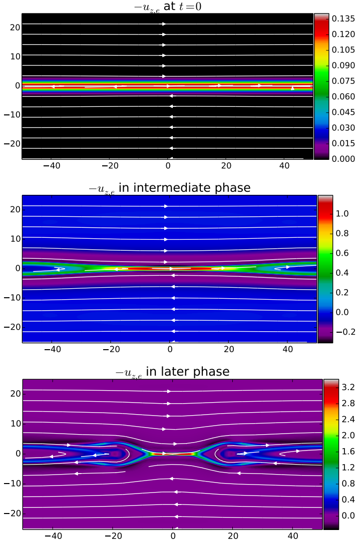
- 10-moment model requires $\partial_m Q_{ijm}$: $\partial_t P_{ij} + \cdots = -\partial_m Q_{ijm}$
- Hammett and Perkins (1990): Find a heat flux approximation so that the linear responses from the multi-fluid moment equations match those from Vlasov equations in the Fourier transformed space (k)
  - 1D in k-space: $\tilde{q}_k = -n_0 \chi_1 \frac{\sqrt{2}}{|k|} i k v_t \tilde{T}_k$
  - Maths are skipped here
- Implications of the heat flux approximation
  - It is collisionless
  - It captures kinetic physics like Landau damping
  - It argues that heat flux is driven by nonlocal tempearture differences
- Further simplifications we made for our real space simulations:
  - replace the varying k by a constant, characteristic k_0
  - replace global nonlocal averages with semi-local averages

$$\Rightarrow \partial_m \tilde{Q}_{ijm} = v_t \left| k_0 \right| \left(P_{ij} - p\delta_{ij} \right)$$

Outline

Comparative Study of a Modified GEM Problem

- PIC (fully kinetic "golden standard") vs. 5- and 10-moment
  - codes: Particle Simulation Code (PSC) vs. Gkeyll
- $100d_{i0} \times 50d_{i0}$, $T_i/T_e = 5$, $n_b/n_0 = 0.3$
- Two 10-moment runs:
  1. $k_{e0} = k_{i0} = 1/10^{-4}d_{e0} \to \infty$ leads to $\mathbf{P}_s \to p_s\hat{\mathbf{I}}$? (otherwise $v_t\,|k_0|\,(P_{ij} - p\delta_{ij})$ blows up)
  2. $k_{e0} = k_{i0} = 1/d_{e0}$ captures key reconnection physics? (since d_{e0} is a critical scale)

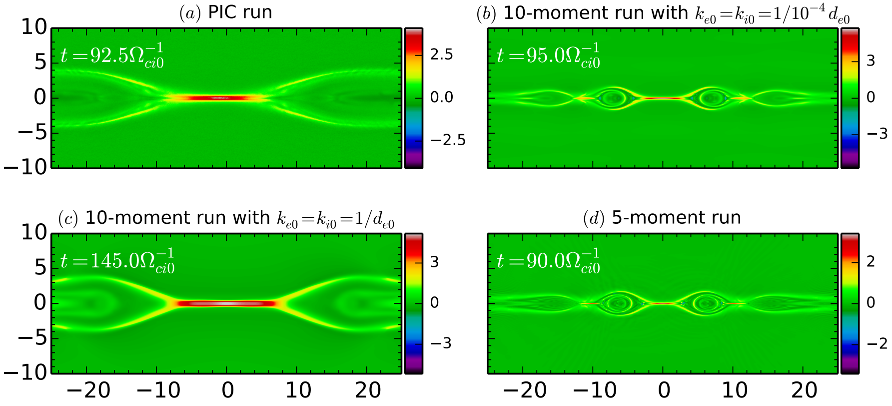


Snapshots of magnetic field lines (white contours) overlaid on out-of-plane electron velocity (color coded) from the 10-moment run with $k_{e0} = k_{i0} = 1/d_{e0}$

Elctron Current Layer Structure

10-moment run with $k_{e0} = k_{i0} = 1/d_{e0}$ agrees with PIC run remarkably well

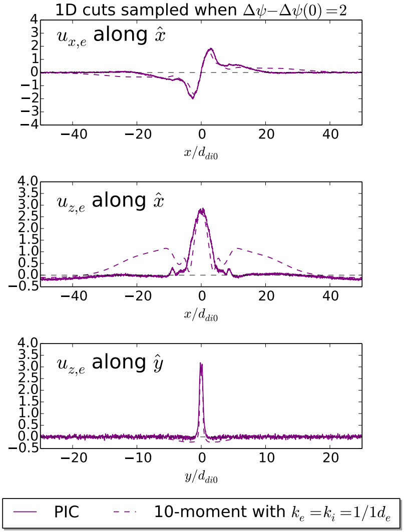
Out-of-plane electron velocity, $u_{z,e}$, when $\Delta\psi - \Delta\psi(t=0) = 2.5$



Note: Only central portions ($50d_{i0} \times 20d_{i0}$) of the domains ($100d_{i0} \times 50d_{i0}$) are shown

1D Cuts of Flow Velocities

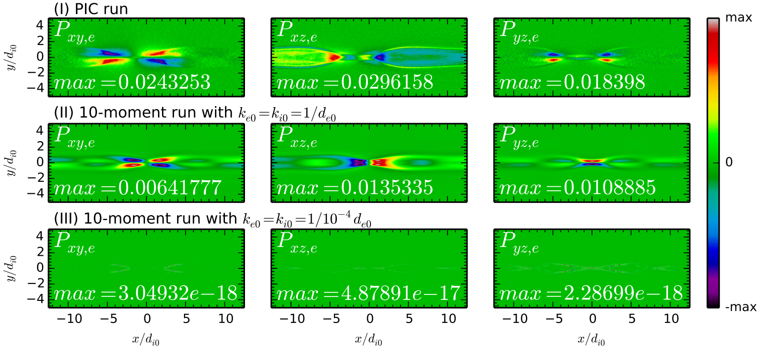
Again, 10-moment run with $k_{e0} = k_{i0} = 1/d_{e0}$ agrees with PIC run remarkably well



Such good agreement persists to later times

Off-Diagonal Elements of $\mathbf{P}_e$

When $\Delta\psi - \Delta\psi(t=0) = 1$



Qualitative and quantitative comparison of terms

- 10-moment run with $k_e = k_i = 1/d_{e0}$ (middle row) recovers the polarities shown in the PIC run (top row)
  - Magnitudes near the X-point are also close
- In the 10-moment run with $k_e = k_i = 1/10^{-4} d_{e0}$ (bottom row), these elements vanish!

Outline

Summary I

- Compared to MHD models, the multi-fluid moment models provide a self-consistent, and more complete description of the plasma
  - The 5-moment equations formally reduce to Hall MHD equation if we assume $\varepsilon_0 \to 0$ ($c \to \infty$, $n_e = n_i$), and $m_e/m_i \to 0$
  - The time step constraint due to artificially quadratic whistler dispersion in Hall MHD is eliminated

Summary II

- We suggested the *paradigm* by Hammett and Perkins (1990) to develop *collisionless* closures, and generalized their 1D results to 3D (but with local approximations)
  - Though the resulting equation of $\mathbf{P}_e$ appears somewhat similar to those used by Hesse and Winske (1993) and Yin et al. (2001), but they did not give a rigorous paradigm, but a plausible explanation
- With the suggested closure (and appropriate parameters), the 10-moment model can capture some key kinetic features in a large scale GEM challenge problem
  - Lower moments like flow velocities agree remarkably well with fully kinetic PIC results
  - Higher moments like pressure tensor terms also agree reasonably well with PIC results

Future Works

Improving the closure

- Rigorous generalization to 3D
- Testing fast algorithms of the nonlocal integration

Application to Earth's magnetosphere

- Promising features:
  - Self-consistent treatments of important non-MHD effects like full $\mathbf{P}_e$ tensor and Hall effect
  - Relaxed time step constraint with simple algorithms
  - Straightforward multi-species handling

Thank you!

References I

Hammett, G. and Perkins, F. (1990). Fluid moment models for Landau damping with application to the ion-temperature-gradient instability. *Physical Review Letters*, 64(25):3019–3022.

Hesse, M. and Winske, D. (1993). Hybrid simulations of collisionless ion tearing. *Geophysical Research Letters*, 20(12):1207–1210.

Le, A., Egedal, J., Daughton, W., Fox, W., and Katz, N. (2009). Equations of state for collisionless guide-field reconnection. *Physical Review Letters*, 102(8):085001.

Ø ieroset, M., Phan, T., Fujimoto, M., Lin, R., and Lepping, R. (2001). In situ detection of collisionless reconnection in the Earth's magnetotail. *Nature*, 412(6845):414–7.

Sullivan, B. P., Bhattacharjee, A., and Huang, Y.-M. (2009). Extension of the electron dissipation region in collisionless Hall magnetohydrodynamics reconnection. *Physics of Plasmas*, 16(10):102111.

References II

Yin, L., Winske, D., Gary, S. P., and Birn, J. (2001). Hybrid and Hall-MHD simulations of collisionless reconnection: Dynamics of the electron pressure tensor. *Journal of Geophysical Research*, 106(A6):10761.